Mark Scheme (Results)

January 2023

Pearson Edexcel International Advanced Level
In Further Pure Mathematics F2 (WFM02)
Paper 01

Question Number	Scheme	Notes	Marks
1(a)	$y=\ln(5+3x)\Rightarrow\frac{dy}{dx}=\frac{3}{5+3x}$	Correct first derivative	B1
	$\frac{dy}{dx}=\frac{3}{5+3x}\Rightarrow\frac{d^2y}{dx^2}=-\frac{9}{(5+3x)^2}\Rightarrow\frac{d^3y}{dx^3}=\frac{54}{(5+3x)^3}$ **M1**: Continues the process of differentiating and reaches $\frac{d^3y}{dx^3}=\frac{k}{(5+3x)^3}$ oe Note this may be achieved via the quotient rule e.g. $\frac{d^3y}{dx^3}=\frac{-9\times-2\times3(5+3x)}{(5+3x)^4}$ **A1**: Correct **simplified** third derivative. Allow e.g. $\frac{54}{(5+3x)^3}$ or $54(5+3x)^{-3}$.		M1 A1
			(3)
(b)	$y_0=\ln5, y_0'=\frac{3}{5}, y_0''=-\frac{9}{25}, y_0'''=\frac{54}{125}$ $\Rightarrow\ln(5+3x)\approx\ln5+\frac{3}{5}x-\frac{9}{25}\frac{x^2}{2!}+\frac{54}{125}\frac{x^3}{3!}+...$ Attempts all values at x = 0 and applies Maclaurin's theorem. Evidence for attempting the values can be taken from at least 2 terms. The form of the expansion must be correct including the factorials or their values. Note that this is "Hence" and so do not allow other methods e.g. Formula Book.		M1
	$\ln(5+3x)\approx\ln5+\frac{3}{5}x-\frac{9}{50}x^2+\frac{9}{125}x^3+...$ Correct expansion. The "ln(5 + 3x) =" is not required.		A1
			(2)
(c)	$\ln(5-3x)\approx\ln5-\frac{3}{5}x-\frac{9}{50}x^2-\frac{9}{125}x^3+...$ Correct expansion even if obtained "from scratch" **OR** for a correct follow through with signs changed on the coefficients of the odd powers of x only in an expansion of the correct form e.g. a polynomial in ascending powers of x.		B1ft
			(1)
(d)	$\ln\frac{(5+3x)}{(5-3x)}=\ln(5+3x)-\ln(5-3x)$ $\ln5+\frac{3}{5}x-\frac{9}{50}x^2+\frac{9}{125}x^3+...-\left(\ln5-\frac{3}{5}x-\frac{9}{50}x^2-\frac{9}{125}x^3+...\right)$ Subtracts **<u>their</u>** 2 **different** series to obtain at least 2 non-zero terms in ascending powers of x.		M1
	$=\frac{6}{5}x+\frac{18}{125}x^3+...$ Correct terms. Allow e.g. $0+\frac{6}{5}x+0x^2+\frac{18}{125}x^3+...$		A1

	Allow both marks to score in (d) provided the **correct** series have been obtained in (b) and (c) by **any** means.		
			(2)
			Total 8

Question Number	Scheme	Notes	Marks
2(a)	$\frac{1}{(2n-1)(2n+1)(2n+3)} \equiv \frac{A}{2n-1}+\frac{B}{2n+1}+\frac{C}{2n+3}$ $\Rightarrow A=...,B=...,C=...$ Correct partial fraction attempt to obtain values for A, B and C		M1
	$\frac{1}{8(2n-1)}-\frac{1}{4(2n+1)}+\frac{1}{8(2n+3)}$ or e.g $\frac{1}{16n-8}-\frac{1}{8n+4}+\frac{1}{16n+24}$ or e.g. $\frac{\frac{1}{8}}{(2n-1)}-\frac{\frac{1}{4}}{(2n+1)}+\frac{\frac{1}{8}}{(2n+3)}$ Correct **partial fractions**. (May be seen in (b)) This mark is **not** for the correct values of A, B and C, it is for the correct fractions.		A1
			(2)
(b)	$\frac{1}{8}\sum_{r=1}^{n}\left(\frac{1}{2r-1}-\frac{2}{2r+1}+\frac{1}{2r+3}\right)=$ $\frac{1}{8}\left(\frac{1}{1}-\frac{2}{3}+\frac{1}{5}\right.$ $+\frac{1}{3}-\frac{2}{5}+\frac{1}{7}$ $+\frac{1}{5}-\frac{2}{7}+\frac{1}{9}$ . . $+\frac{1}{2n-3}-\frac{2}{2n-1}+\frac{1}{2n+1}$ $\left.+\frac{1}{2n-1}-\frac{2}{2n+1}+\frac{1}{2n+3}\right)$ Uses the method of differences to find sufficient terms to establish cancelling. E.g. 3 rows at the start and 2 rows at the end or vice versa This may be implied if they extract the correct non-cancelling terms.		M1
	$=\frac{1}{8}\left(1-\frac{1}{3}-\frac{1}{2n+1}+\frac{1}{2n+3}\right)$ or e.g. $=\frac{1}{8}\left(1-\frac{2}{3}+\frac{1}{3}+\frac{1}{2n+1}-\frac{2}{2n+1}+\frac{1}{2n+3}\right)$ Identifies the correct non-cancelling terms. May be unsimplified.		A1
	$=\frac{1}{8}\left(\frac{2(2n+1)(2n+3)-3(2n+3)+3(2n+1)}{3(2n+1)(2n+3)}\right)=...$ Attempts to combine terms into one fraction. There must have been at least one constant term and at least 2 different algebraic terms with at least 3 terms in the numerator when combining the fractions.		dM1

	$= \dfrac{n(n+2)}{3(2n+1)(2n+3)}$	Cao	A1
			(4)
			Total 6

Question Number	Scheme	Notes	Marks
3(a)	$x^2\frac{dy}{dx}+xy=2y^2 \qquad y=\frac{1}{z}$		
	$\frac{dy}{dx}=-\frac{1}{z^2}\frac{dz}{dx}$	Correct differentiation	B1
	$-\frac{x^2}{z^2}\frac{dz}{dx}+\frac{x}{z}=\frac{2}{z^2}$	Substitutes into the given differential equation	M1
	$\frac{dz}{dx}-\frac{z}{x}=-\frac{2}{x^2}$ *	Achieves the printed answer with no errors. Allow this to be written down following a correct substitution i.e. with no intermediate step.	A1*
			(3)
(a) Way 2	$y=\frac{1}{z}\Rightarrow zy=1\Rightarrow y\frac{dz}{dx}+z\frac{dy}{dx}=0$	Correct differentiation	B1
	$-\frac{y}{z}x^2\frac{dz}{dx}+\frac{x}{z}=\frac{2}{z^2}$	Substitutes into the given differential equation	M1
	$\frac{dz}{dx}-\frac{z}{x}=-\frac{2}{x^2}$ *	Achieves the printed answer with no errors. Allow this to be written down following a correct substitution i.e. with no intermediate step.	A1*
(a) Way 3	$y=\frac{1}{z}\Rightarrow z=\frac{1}{y}\Rightarrow\frac{dz}{dx}=-\frac{1}{y^2}\frac{dy}{dx}$	Correct differentiation	B1
	$-\frac{1}{y^2}\frac{dy}{dx}-\frac{1}{xy}=-\frac{2}{x^2}$	Substitutes into differential equation (II)	M1
	$x^2\frac{dy}{dx}+xy=2y^2$	Obtains differential equation (I) with no errors. Allow this to be written down following a correct substitution i.e. with no intermediate step.	A1*
(b)	$I=e^{-\int\frac{1}{x}dx}=e^{-\ln x}=\frac{1}{x}$	Correct integrating factor of $\frac{1}{x}$	B1
	$\frac{z}{x}=-\int\frac{2}{x^3}dx$	For $Iz=-\int\frac{2I}{x^2}dx$. Condone the "dx" missing.	M1
	$\frac{z}{x}=\frac{1}{x^2}+c$	Correct equation including constant	A1
	$z=\frac{1}{x}+cx$	Correct equation in the required form	A1
			(4)
(c)	$\frac{1}{y}=\frac{1}{x}+cx\Rightarrow-\frac{8}{3}=\frac{1}{3}+3c\Rightarrow c=-1$	Reverses the substitution and uses the given conditions to find their constant	M1

	$\frac{1}{y}=\frac{1}{x}-x \Rightarrow y=\frac{x}{1-x^2}$	Correct equation for y in terms of x. Allow any correct equivalents e.g. $y=\frac{1}{x^{-1}-x}$, $y=\frac{1}{\frac{1}{x}-x}$	A1
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
4(a)	$\frac{dy}{dx}=y^2-x\Rightarrow\frac{d^2y}{dx^2}=2y\frac{dy}{dx}-1$	Correct expression for $\frac{d^2y}{dx^2}$	B1
	$\frac{d^3y}{dx^3}=2y\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^2$ **M1**: Applies the product rule to obtain $\frac{d^3y}{dx^3}=Ay\frac{d^2y}{dx^2}+\ldots$ or $\frac{d^3y}{dx^3}=\ldots+B\left(\frac{dy}{dx}\right)^2$ where … is non-zero **A1**: Correct expression. Apply isw if necessary.		M1 A1
	$\frac{d^3y}{dx^3}=2y\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^2\Rightarrow\frac{d^4y}{dx^4}=2y\frac{d^3y}{dx^3}+2\frac{dy}{dx}\frac{d^2y}{dx^2}+4\frac{dy}{dx}\frac{d^2y}{dx^2}$ $\frac{d^4y}{dx^4}=2y\frac{d^3y}{dx^3}+6\frac{dy}{dx}\frac{d^2y}{dx^2}$ Correct expression for $\frac{d^4y}{dx^4}$ or correct values for *A* and *B*.		A1
	<u>**Note:**</u> If e.g. $\frac{d^2y}{dx^2}=2y\frac{dy}{dx}$ is obtained, allow recovery in (a) so B0M1A1A1 is possible.		
			(4)
(b)	$(y)_{-1}=1, (y')_{-1}=2, (y'')_{-1}=3, (y''')_{-1}=14, (y'''')_{-1}=64$ Attempts the values up to at least the 3rd derivative using $x=-1$ and *y* = 1 Condone slips provided the intention is clear. May be implied by their values.		M1
	$(y=)1+2(x+1)+\frac{3(x+1)^2}{2}+\frac{14(x+1)^3}{3!}+\frac{64(x+1)^4}{4!}+\ldots$ Correct application of the Taylor series in powers of (*x* + 1) If the expansion is just written down with no formula quoted then it must be correct for their values. E.g. *y* = –1 + … with no evidence *y* = f (–1) was meant scores M0		M1
	$(y=)1+2(x+1)+\frac{3(x+1)^2}{2}+\frac{7(x+1)^3}{3}+\frac{8(x+1)^4}{3}+\ldots$ Correct simplified expansion. The "*y* =" is not required.		A1
			(3)
			Total 7

Question 5
General Guidance

B1: This mark is for sight of –8 seen as part of their working. It may be seen as e.g. embedded in an inequality, as part of their solution if they consider for example $x > -8$, $x < -8$ or –8 is seen in a sketch etc.
Do not allow for just e.g. $x + 8 > 0$,

M1: Any valid attempt to find at least one critical value other than $x = -8$ (see below).
Condone use of e.g. "=", ">", "<" etc as part of their working.
Note these usually come in pairs as $3, -\frac{19}{3}$ or $3, -13$

M1: A valid attempt to find all critical values.
Condone use of e.g. "=", ">", "<" etc as part of their working.

A1: Any 2 critical values other than $x = -8$. May be seen embedded in an inequality or on a sketch.

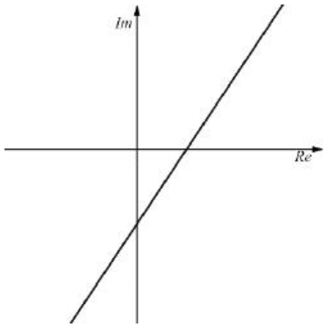
A1: 2 correct regions

A1: All correct with no extra regions

Question Number	Scheme	Notes	Marks
5	$(x=)-8$	This cv stated or used	B1
	For cv's $3, -\frac{19}{3}$ **OR** For cv's $3, -13$		
	Examples: $x^2-9=(x+8)(6-2x)\Rightarrow x=...$ **or** $(x^2-9)(x+8)=(x+8)^2(6-2x)\Rightarrow x=...$ **or** $\frac{x^2-9}{(x+8)}-(6-2x)=0\Rightarrow x=...>$ NB leads to $3x^2+10x-57=0$	Examples: $x^2-9=-(x+8)(6-2x)\Rightarrow x=...$ **or** $-(x^2-9)(x+8)=(x+8)^2(6-2x)\Rightarrow x=...$ **or** $\frac{x^2-9}{-(x+8)}-(6-2x)=0\Rightarrow x=...>$ NB leads to $x^2+10x-39=0$	M1
	For cv's $3, -\frac{19}{3}$ **AND** For cv's $3, -13$		
	Examples: $x^2-9=(x+8)(6-2x)\Rightarrow x=...$ **or** $(x^2-9)(x+8)=(x+8)^2(6-2x)\Rightarrow x=...$ **or** $\frac{x^2-9}{(x+8)}-(6-2x)=0\Rightarrow x=...>$ NB leads to $3x^2+10x-57=0$	Examples: $x^2-9=-(x+8)(6-2x)\Rightarrow x=...$ **or** $-(x^2-9)(x+8)=(x+8)^2(6-2x)\Rightarrow x=...$ **or** $\frac{x^2-9}{-(x+8)}-(6-2x)=0\Rightarrow x=...>$ NB leads to $x^2+10x-39=0$	M1
	Any two of: $x=-13, -\frac{19}{3}, 3$	For any two of these cv's. May be seen embedded in their inequalities. **Depends on at least one previous M mark.**	A1
	$-13<x<-8,\ -8<x<-\frac{19}{3},\ x>3$ **A1**: Any 2 of these inequalities. Note that $-13<x<-\frac{19}{3},\ x\neq-8$ would count as 2 correct inequalities. Also condone $-13<x<-\frac{19}{3},\ \ x>3$ as 2 correct inequalities. **Depends on at least one previous M mark.** **A1**: All correct and no other regions. **Depends on all previous marks.**		A1 A1
	Allow equivalent notation for the inequalities e.g. for $-13<x<-8$ allow $x>-13$ and $x<-8,\ x>-13,\ x<-8,\ -8>x>-13, (-13,-8),\ \{x: x>-13\cap x<-8\}$ But not $x>-13$ or $x<-8$ Note that $-13<x<-\frac{19}{3},\ x\neq-8,\ x>3$ is fully correct.		
			(6)
			Total 6

Note that it is possible to find all the cv's by squaring both sides of the equation:

	$(x=)-8$	This cv stated or used	B1
	$\frac{(x^2-9)^2}{(x+8)^2}=(6-2x)^2 \Rightarrow x^4-18x^2+81=(36-24x+4x^2)(x^2+16x+64)$ $\Rightarrow 3x^4+40x^3-74x^2-960x+2223=0 \Rightarrow x=...$ M2 Requires a complete attempt to square both sides, multiply up to obtain a quartic equation and an attempt to solve to find at least 1 critical value other than $x = -8$		M1M1
	Any two of: $x=-13,\ -\frac{19}{3},\ 3$	For any two of these cv's. May be seen embedded in their inequalities. **Depends on both previous M marks.**	A1
	$-13<x<-8,\ \ -8<x<-\frac{19}{3},\ \ x>3$ **A1**: Any 2 of these inequalities. Note that $-13<x<-\frac{19}{3},\ \ x\neq-8$ would count as 2 correct inequalities. Also condone $-13<x<-\frac{19}{3},\ \ x>3$ as 2 correct inequalities. **Depends on at least one previous M mark.** **A1**: All correct and no other regions. **Depends on all previous marks.**		A1 A1
	Allow equivalent notation for the inequalities e.g. for $-13<x<-8$ allow $x>-13$ and $x<-8$, $x>-13,\ x<-8,\ -8>x>-13, (-13,-8), \{x: x>-13 \cap x<-8\}$ But not $x>-13$ or $x<-8$ Note that $-13<x<-\frac{19}{3},\ \ x\neq-8,\ \ x>3$ is fully correct.		

Question Number	Scheme	Notes	Marks
6(a)		A straight line anywhere that is not vertical or horizontal which does not pass through the origin. It may be solid or dotted. Clear "V" shapes score M0.	M1
		A straight line in the correct position. Must have a positive gradient and lie in quadrants 1, 3 and 4. Ignore any intercepts correct or incorrect. If there are other lines that are clearly "construction" lines e.g. a line from 2i to 3 they can be ignored. The line may be solid or dotted. However, if there are clearly several lines then score A0.	A1
			(2)

<u>Part (b)</u>

The approaches below are the ones that have been seen most often.
Apply the mark scheme to the overall method the candidate has chosen.
There may be several attempts:

- **If none are crossed out, mark all attempts and score the best single complete attempt**
- **If some attempts are crossed out, mark the uncrossed out work**
- **If everything is crossed out, mark all the work and score the best single complete attempt**

Note that the question does not specify the variables the candidates should work in so they may use: e.g. $z = x + \mathrm{i}y$ and $w = u + \mathrm{i}v$ or $w = x + \mathrm{i}y$ and $z = u + \mathrm{i}v$ or any other letters so please check the work carefully.

Note that the M marks are all dependent on each other.

(b) **Way 1**	$w=\frac{\mathrm{i}z}{z-2\mathrm{i}} \Rightarrow z=\frac{2w\mathrm{i}}{w-\mathrm{i}}$	Attempts to make z the subject. Must obtain the form $\frac{aw\mathrm{i}}{bw+c\mathrm{i}}$, a, b, c real and non-zero.	M1
	$z=\frac{2(u+\mathrm{i}v)\mathrm{i}}{u+\mathrm{i}v-\mathrm{i}}$ or e.g. $z=\frac{2(x+\mathrm{i}y)\mathrm{i}}{x+\mathrm{i}y-\mathrm{i}}$ $z=\frac{2(u+\mathrm{i}v)\mathrm{i}}{u+(v-1)\mathrm{i}}\times\frac{u-(v-1)\mathrm{i}}{u-(v-1)\mathrm{i}}$ or equivalent Introduces $w=u+\mathrm{i}v$ or e.g. $w=x+\mathrm{i}y$ and attempts to multiply numerator and denominator by the complex conjugate of the denominator. The above statement would be sufficient e.g. no expansion is needed for this mark.		**d**M1
	$z=\frac{-2u}{u^2+(v-1)^2}+\frac{2u^2+2v(v-1)}{u^2+(v-1)^2}\mathrm{i}$ or e.g. $z=\frac{-2x}{x^2+(y-1)^2}+\frac{2x^2+2y(y-1)}{x^2+(y-1)^2}\mathrm{i}$ or $z=\frac{-2uv+2u(v-1)+\left(2u^2+2v(v-1)\right)\mathrm{i}}{u^2+(v-1)^2}$ or e.g. $z=\frac{-2xy+2x(y-1)+\left(2x^2+2y(y-1)\right)\mathrm{i}}{x^2+(y-1)^2}$ Correct expression for z in terms of their variables with real and imaginary parts identified. May be embedded as above or stated explicitly.		A1
	$\lvert z-2\mathrm{i}\rvert=\lvert z-3\rvert \Rightarrow y-1=\frac{3}{2}\left(x-\frac{3}{2}\right)\left(y=\frac{3}{2}x-\frac{5}{4},\ 6x-4y=5\right)$ $\Rightarrow \frac{2u^2+2v(v-1)}{u^2+(v-1)^2}-1=\frac{3}{2}\left(\frac{-2u}{u^2+(v-1)^2}-\frac{3}{2}\right)$ Attempts the Cartesian equation of the locus of z and substitutes for x and y or equivalent using their variables to obtain an equation in u and v (or their variables). Condone slips with the locus of z but must be a linear equation in any form but with a non-zero constant term. **or** $\lvert z-2\mathrm{i}\rvert=\lvert z-3\rvert \Rightarrow \left\lvert\frac{-2u}{u^2+(v-1)^2}+\frac{2u^2+2v(v-1)}{u^2+(v-1)^2}\mathrm{i}-2\mathrm{i}\right\rvert=\left\lvert\frac{-2u}{u^2+(v-1)^2}+\frac{2u^2+2v(v-1)}{u^2+(v-1)^2}\mathrm{i}-3\right\rvert$ $\Rightarrow\left(\frac{-2u}{u^2+(v-1)^2}\right)^2+\left(\frac{2u^2+2v(v-1)}{u^2+(v-1)^2}-2\right)^2=\left(\frac{-2u}{u^2+(v-1)^2}-3\right)^2+\left(\frac{2u^2+2v(v-1)}{u^2+(v-1)^2}\right)^2$ Substitutes their z into the locus of z and applies Pythagoras correctly to obtain an equation in u and v (or their variables). Note that here, further progress is unlikely.		**dd**M1
	$13u^2+13v^2+12u-18v+5=0 \Rightarrow u^2+v^2+\frac{12}{13}u-\frac{18}{13}v+\frac{9}{13}=\frac{4}{13}$ $\Rightarrow\left(u+\frac{6}{13}\right)^2+\left(v-\frac{9}{13}\right)^2=\frac{4}{13}$ Attempts to complete the square on their equation in u and v where u^2 and v^2 have the same coefficient. Award for e.g. $u^2+v^2+\alpha u+\beta v+...=\left(u+\frac{\alpha}{2}\right)^2+\left(v+\frac{\beta}{2}\right)^2+...=...$		**ddd**M1

	Attempts using the form $u^2+v^2+2gu+2fv+c=0$ send to review.		
	$\left\|w-\left(-\frac{6}{13}+\frac{9}{13}\mathrm{i}\right)\right\|=\frac{2}{\sqrt{13}}$	Correct equation in the required form	A1
			Total 8

(b) **Way 2**	$w=\frac{\mathrm{i}z}{z-2\mathrm{i}}\Rightarrow z=\frac{2w\mathrm{i}}{w-\mathrm{i}}$	Attempts to make z the subject. Must obtain the form $\frac{aw\mathrm{i}}{bw+c\mathrm{i}}$, a, b, c real and non-zero.	M1
	$\|z-2\mathrm{i}\|=\|z-3\|\Rightarrow\left\|\frac{2w\mathrm{i}}{w-\mathrm{i}}-2\mathrm{i}\right\|=\left\|\frac{2w\mathrm{i}}{w-\mathrm{i}}-3\right\|$ $\Rightarrow\left\|\frac{2w\mathrm{i}-2w\mathrm{i}-2}{w-\mathrm{i}}\right\|=\left\|\frac{2w\mathrm{i}-3w+3\mathrm{i}}{w-\mathrm{i}}\right\|$ Introduces z in terms of w into the given locus and attempts to combine terms		**d**M1
	$\left\|\frac{-2}{w-\mathrm{i}}\right\|=\left\|\frac{2w\mathrm{i}-3w+3\mathrm{i}}{w-\mathrm{i}}\right\|\Rightarrow\|-2\|=\|2w\mathrm{i}-3w+3\mathrm{i}\|$ Correct equation with fractions removed		A1
	$\|2(u+\mathrm{i}v)\mathrm{i}-3(u+\mathrm{i}v)+3\mathrm{i}\|=2\Rightarrow(3u+2v)^2+(3v-2u-3)^2=4$ Introduces e.g. $w = u + \mathrm{i}v$ and applies Pythagoras correctly		**dd**M1
	$13u^2+13v^2+12u-18v+9=4\Rightarrow u^2+v^2+\frac{12}{13}u-\frac{18}{13}v+\frac{9}{13}=\frac{4}{13}$ $\Rightarrow\left(u+\frac{6}{13}\right)^2+\left(v-\frac{9}{13}\right)^2=\frac{4}{13}$ Attempts to complete the square on their equation in u and v where u^2 and v^2 have the same coefficient. Award for e.g. $u^2+v^2+\alpha u+\beta v+...=\left(u+\frac{\alpha}{2}\right)^2+\left(v+\frac{\beta}{2}\right)^2+...=...$ Attempts using the form $u^2+v^2+2gu+2fv+c=0$ send to review.		**ddd**M1
	$\left\|w-\left(-\frac{6}{13}+\frac{9}{13}\mathrm{i}\right)\right\|=\frac{2}{\sqrt{13}}$	Correct equation in the required form	A1
			Total 8

(b) Way 3	$w=\frac{\mathrm{i}z}{z-2\mathrm{i}} \Rightarrow z=\frac{2w\mathrm{i}}{w-\mathrm{i}}$	Attempts to make z the subject. Must obtain the form $\frac{aw\mathrm{i}}{bw+c\mathrm{i}}$, a, b, c real and non-zero.	M1
	$\lvert z-2\mathrm{i}\rvert=\lvert z-3\rvert \Rightarrow \left\lvert\frac{2w\mathrm{i}}{w-\mathrm{i}}-2\mathrm{i}\right\rvert=\left\lvert\frac{2w\mathrm{i}}{w-\mathrm{i}}-3\right\rvert$ $\Rightarrow\left\lvert\frac{2w\mathrm{i}-2w\mathrm{i}-2}{w-\mathrm{i}}\right\rvert=\left\lvert\frac{2w\mathrm{i}-3w+3\mathrm{i}}{w-\mathrm{i}}\right\rvert$ Introduces z and attempts to combine terms		**d**M1
	$\left\lvert\frac{-2}{w-\mathrm{i}}\right\rvert=\left\lvert\frac{2w\mathrm{i}-3w+3\mathrm{i}}{w-\mathrm{i}}\right\rvert \Rightarrow \lvert -2\rvert=\lvert 2w\mathrm{i}-3w+3\mathrm{i}\rvert$ Correct equation with fractions removed		A1
	$\lvert w(2\mathrm{i}-3)+3\mathrm{i}\rvert=\left\lvert(2\mathrm{i}-3)\left(w+\frac{3\mathrm{i}}{2\mathrm{i}-3}\right)\right\rvert=\lvert 2\mathrm{i}-3\rvert\left\lvert w+\frac{6-9\mathrm{i}}{13}\right\rvert=2$ Attempts to isolate w and rationalise denominator of other term		**dd**M1
	$\sqrt{13}\left\lvert w-\left(-\frac{6}{13}+\frac{9}{13}\mathrm{i}\right)\right\rvert=2 \Rightarrow \left\lvert w-\left(-\frac{6}{13}+\frac{9}{13}\mathrm{i}\right)\right\rvert=\frac{2}{\sqrt{13}}$ M1: Completes the process by dividing by their $\lvert 2\mathrm{i}-3\rvert$ A1: Correct equation in the required form		**ddd**M1A1
			(6)

Question Number	Scheme	Notes	Marks
7(a)	Condone use of e.g. C + iS for $\cos x+\text{i}\sin x$ if the intention is clear.		
	$(\cos 5x\equiv)\text{Re}(\cos x+\text{i}\sin x)^5\equiv\cos^5 x+\binom{5}{2}\cos^3 x(\text{i}\sin x)^2+\binom{5}{4}\cos x(\text{i}\sin x)^4$ Identifies the correct terms of the binomial expansion of $(\cos x+\text{i}\sin x)^5$ They may expand $(\cos x+\text{i}\sin x)^5$ completely but there must be an attempt to extract the real terms which must have the correct binomial coefficients combined with the correct powers of sin x and cos x. Condone use of a different variable e.g. θ.		M1
	$(\cos 5x\equiv)\cos^5 x-10\cos^3 x\sin^2 x+5\cos x\sin^4 x$ Correct simplified expression. Condone use of a different variable e.g. θ.		A1
	$\equiv\cos x\left(\cos^4 x-10\cos^2 x\sin^2 x+5\sin^4 x\right)$ $\equiv\cos x\left(\left(1-\sin^2 x\right)^2-10\left(1-\sin^2 x\right)\sin^2 x+5\sin^4 x\right)$ Applies $\cos^2 x=1-\sin^2 x$ to obtain an expression in terms of sin x inside the bracket. Condone use of a different variable e.g. θ.		M1
	$\equiv\cos x\left(16\sin^4 x-12\sin^2 x+1\right)$	Correct expression. Must be in terms of x now. The "cos5x =" is not required.	A1
			(4)
(b)	Allow use of a different variable in (b) e.g. x for <u>all</u> marks.		
	$\cos 5\theta=\sin 2\theta\sin\theta-\cos\theta$ $\Rightarrow\cos\theta\left(16\sin^4\theta-12\sin^2\theta+1\right)=2\sin^2\theta\cos\theta-\cos\theta$ $\Rightarrow\cos\theta\left(16\sin^4\theta-14\sin^2\theta+2\right)=0$ Uses the result from part (a) with $\sin 2\theta=2\sin\theta\cos\theta$ and collects terms		M1
	$16\sin^4\theta-14\sin^2\theta+2=0$ $\Rightarrow\sin^2\theta=\frac{7\pm\sqrt{17}}{16}\Rightarrow\sin\theta=...$ Solves for $\sin^2\theta$ by any method including calculator and takes square root to obtain at least one value for $\sin\theta$. Depends on the first mark. May be implied by their values of $\sin\theta$ or θ. NB $\frac{7\pm\sqrt{17}}{16}=0.69519...,\ 0.17980...$		dM1
	$\sin\theta=\sqrt{\frac{7\pm\sqrt{17}}{16}}\Rightarrow\theta=...$ NB $\sqrt{\frac{7\pm\sqrt{17}}{16}}=0.833783...,\ \ 0.424035...$ A full method to reach at least one value for θ. Depends on the previous mark. May be implied by their values of θ		ddM1
	$(\theta=)0.986,\ 0.438$	Correct values and no others in range. Allow awrt these values.	A1
			(4)
			Total 8

<u>Note that it is possible to do 7(b) by changing to cos *θ* e.g.</u>

$$\cos\theta\left(16\sin^4\theta - 12\sin^2\theta + 1\right) = \cos\theta\left(16\left(1-\cos^2\theta\right)^2 - 12\left(1-\cos^2\theta\right)+1\right)$$

$$\cos\theta\left(16\left(1-\cos^2\theta\right)^2 - 12\left(1-\cos^2\theta\right)+1\right) = 2\sin^2\theta\cos\theta - \cos\theta$$

$$16\cos^4\theta - 18\cos^2\theta + 4 = 0$$

$$\cos^2\theta = \frac{9\pm\sqrt{17}}{16} \Rightarrow \cos\theta = \sqrt{\frac{9\pm\sqrt{17}}{16}}$$

$$(\theta =)0.986,\ 0.438$$

This is acceptable as they used part (a) and can be scored as:

M1: Uses part (a) with $\sin^2\theta = 1-\cos^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$ and collects terms.

dM1: Solves for $\cos^2\theta$ by any method including calculator and takes square root to obtain at least one value for $\cos\theta$. Depends on the first mark. May be implied by their values of $\cos\theta$ or θ.

NB $\frac{9\pm\sqrt{17}}{16} = 0.82019...,\ 0.30480...$

dM1: A full method to reach at least one value for θ.
Depends on the previous mark. May be implied by their values of θ

NB $\sqrt{\frac{9\pm\sqrt{17}}{16}} = 0.905645...,\ 0.552092...$

A1: $(\theta =)0.986,\ 0.438$

Question Number	Scheme	Notes	Marks
8(a)	$y = r\sin\theta = (1-\sin\theta)\sin\theta = \sin\theta - \sin^2\theta$ $\Rightarrow \dfrac{dy}{d\theta} = \cos\theta - 2\sin\theta\cos\theta$ or e.g. $\Rightarrow \dfrac{dy}{d\theta} = \cos\theta - \sin 2\theta$	Differentiates $(1-\sin\theta)\sin\theta$ to achieve $\pm\cos\theta \pm k\sin\theta\cos\theta$ or equivalent. Use of $y = r\cos\theta$ or $x = r\cos\theta$ scores M0	M1
		Correct derivative in any form.	A1
	$\cos\theta - 2\sin\theta\cos\theta = 0 \Rightarrow \cos\theta(1-2\sin\theta) = 0 \Rightarrow \sin\theta = \dfrac{1}{2} \Rightarrow \theta = ...$ Solves to find a value for θ. Depends on the first M.		dM1
	$\left(\dfrac{1}{2}, \dfrac{\pi}{6}\right)$ Correct coordinates and no others. Isw if necessary e.g. if written as $\left(\dfrac{\pi}{6}, \dfrac{1}{2}\right)$ after correct values seen or implied award A1. Allow e.g. $\theta = \dfrac{\pi}{6},\ r = \dfrac{1}{2}$. **The value of *r* must be seen in (a) – i.e. do not allow recovery in (b).**		A1
			(4)
(b) Way 1	Note that the $\dfrac{1}{2}$ in $\dfrac{1}{2}\int r^2\, d\theta$ is not required for the first 4 marks		
	$\int (1-\sin\theta)^2\, d\theta = \int \left(1 - 2\sin\theta + \sin^2\theta\right) d\theta$ $= \int \left(1 - 2\sin\theta + \dfrac{1}{2} - \dfrac{1}{2}\cos 2\theta\right) d\theta$	Attempts $\left(\dfrac{1}{2}\right)\int r^2\, d\theta$ and applies $\sin^2\theta = \pm\dfrac{1}{2} \pm \dfrac{1}{2}\cos 2\theta$	M1
	$\int (1-\sin\theta)^2\, d\theta = \dfrac{3}{2}\theta + 2\cos\theta - \dfrac{1}{4}\sin 2\theta\,(+c)$ Correct integration. Condone mixed variables e.g. $\int (1-\sin\theta)^2\, d\theta = \dfrac{3}{2}x + 2\cos\theta - \dfrac{1}{4}\sin 2\theta\,(+c)$		A1
	$\left(\dfrac{1}{2}\right)\left[\dfrac{3}{2}\theta + 2\cos\theta - \dfrac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{6}} = \left(\dfrac{1}{2}\right)\left[\left(\dfrac{\pi}{4} + \sqrt{3} - \dfrac{\sqrt{3}}{8}\right) - (2)\right]\left(= \dfrac{\pi}{8} + \dfrac{7\sqrt{3}}{16} - 1\right)$ Applies the limits of 0 and their $\dfrac{\pi}{6}$ to their integration. The $\dfrac{1}{2}$ is not required. For the integration look for at least $\pm\int \sin\theta\, d\theta \to \pm\cos\theta$		M1
	Triangle: $\dfrac{1}{2} \times \dfrac{1}{2}\sin\dfrac{\pi}{6} \times \dfrac{1}{2}\cos\dfrac{\pi}{6}\left(= \dfrac{\sqrt{3}}{32}\right)$ Uses a correct strategy for the area of the triangle		M1
	Area of R = $\dfrac{\pi}{8} + \dfrac{7\sqrt{3}}{16} - 1 + \dfrac{\sqrt{3}}{32}$	Fully correct method for the required area. **Depends on all previous method marks.**	dM1

	$\frac{1}{32}\left(4\pi+15\sqrt{3}-32\right)$	Cao	A1
			(6)
			Total 10

	Note that the $\frac{1}{2}$ in $\frac{1}{2}\int r^2\,\mathrm{d}\theta$ is not required for the first 3 marks		
(b) Way 2	$\int(1-\sin\theta)^2\,\mathrm{d}\theta=\int\left(1-2\sin\theta+\sin^2\theta\right)\mathrm{d}\theta$ $=\int\left(1-2\sin\theta+\frac{1}{2}-\frac{1}{2}\cos2\theta\right)\mathrm{d}\theta$	Attempts $\left(\frac{1}{2}\right)\int r^2\,\mathrm{d}\theta$ and applies $\sin^2\theta=\pm\frac{1}{2}\pm\frac{1}{2}\cos2\theta$.	M1
	$\int(1-\sin\theta)^2\,\mathrm{d}\theta=\frac{3}{2}\theta+2\cos\theta-\frac{1}{4}\sin2\theta(+c)$ Correct integration. Condone mixed variables e.g. $\int(1-\sin\theta)^2\,\mathrm{d}\theta=\frac{3}{2}x+2\cos\theta-\frac{1}{4}\sin2\theta(+c)$		A1
	$\left(\frac{1}{2}\right)\left[\frac{3}{2}\theta+2\cos\theta-\frac{1}{4}\sin2\theta\right]_0^{\frac{\pi}{2}}=\left(\frac{1}{2}\right)\left[\left(\frac{3\pi}{4}+0-0\right)-(2)\right]\left(=\frac{3\pi}{8}-1\right)$ Evidence of use of **both** limits 0 and $\frac{\pi}{2}$ to their integration. The $\frac{1}{2}$ is not required. For the integration look for at least $\pm\int\sin\theta\,\mathrm{d}\theta\to\pm\cos\theta$		M1
	Triangle – "Segment": $\frac{1}{2}\times\frac{1}{2}\sin\frac{\pi}{6}\times\frac{1}{2}\cos\frac{\pi}{6}-\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}(1-\sin\theta)^2\,\mathrm{d}\theta$ $\frac{\sqrt{3}}{32}-\frac{1}{2}\left[\frac{3}{2}\theta+2\cos\theta-\frac{1}{4}\sin2\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(=\frac{15\sqrt{3}}{32}-\frac{\pi}{4}\right)$ Uses a fully correct strategy for the area above the curve between *O* and *P*. Requires a correct method for the triangle as in Way 1 and a correct method for the "segment" using **both** their $\frac{\pi}{6}$ and $\frac{\pi}{2}$.		M1
	Area of $R=\frac{3\pi}{8}-1+\frac{15\sqrt{3}}{32}-\frac{\pi}{4}$	Fully correct method for the required area. Depends on all previous method marks.	dM1
	$\frac{1}{32}\left(4\pi+15\sqrt{3}-32\right)$	cao	A1
			(6)

Question Number	Scheme	Notes	Marks
9(a)(i)	$x = t^{\frac{1}{2}} \Rightarrow \frac{dx}{dy} = \frac{1}{2}t^{-\frac{1}{2}}\frac{dt}{dy} \Rightarrow \frac{dy}{dx} = ...$ or $t = x^2 \Rightarrow \frac{dt}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = ...$ Applies the chain rule and proceeds to an expression for $\frac{dy}{dx}$		M1
	$\Rightarrow \frac{dy}{dx} = 2t^{\frac{1}{2}}\frac{dy}{dt}$	Any correct expression for $\frac{dy}{dx}$ in terms of y and t	A1
(a)(ii)	$\frac{dy}{dx} = 2t^{\frac{1}{2}}\frac{dy}{dt} \Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dt}t^{-\frac{1}{2}}\frac{dt}{dx} + 2t^{\frac{1}{2}}\frac{d^2y}{dt^2}\frac{dt}{dx}$ **dM1**: Uses the product rule to differentiate an equation of the form $\frac{dy}{dx} = kt^{\frac{1}{2}}\frac{dy}{dt}$ or equivalent e.g. $\frac{dy}{dx} = kx\frac{dy}{dt}$ to obtain $\frac{d^2y}{dx^2} = \alpha t^{-\frac{1}{2}}\frac{dy}{dt}\frac{dt}{dx} + ...$ or $\frac{d^2y}{dx^2} = ... + \beta t^{\frac{1}{2}}\frac{d^2y}{dt^2}\frac{dt}{dx}$ **<u>or equivalent expressions</u>** where … is non-zero **A1**: <u>Any</u> correct expression for $\frac{d^2y}{dx^2}$		**d**M1A1
	$\frac{dy}{dt}t^{-\frac{1}{2}}\frac{dt}{dx} + 2t^{\frac{1}{2}}\frac{d^2y}{dt^2}\frac{dt}{dx} = \frac{dy}{dt}t^{-\frac{1}{2}} \times 2t^{\frac{1}{2}} + 2t^{\frac{1}{2}}\frac{d^2y}{dt^2}\frac{dt}{dx}$ $\frac{d^2y}{dx^2} = 2\frac{dy}{dt} + 4t\frac{d^2y}{dt^2}$ Correct expression in terms of y and t		A1
			(5)
(b)	$x\frac{d^2y}{dx^2} - (6x^2+1)\frac{dy}{dx} + 9x^3y = x^5 \Rightarrow t^{\frac{1}{2}}\left(2\frac{dy}{dt} + 4t\frac{d^2y}{dt^2}\right) - (6t+1)2t^{\frac{1}{2}}\frac{dy}{dt} + 9t^{\frac{3}{2}}y = t^{\frac{5}{2}}$ Substitutes their expressions from part (a) and replaces x with $t^{\frac{1}{2}}$		M1
	$2t^{\frac{1}{2}}\frac{dy}{dt} + 4t^{\frac{3}{2}}\frac{d^2y}{dt^2} - 12t^{\frac{3}{2}}\frac{dy}{dt} - 2t^{\frac{1}{2}}\frac{dy}{dt} + 9t^{\frac{3}{2}}y = t^{\frac{5}{2}}$ $\Rightarrow 4\frac{d^2y}{dt^2} - 12\frac{dy}{dt} + 9y = t$ * Obtains the given answer with no errors and sufficient working shown – at least one intermediate line after substitution but check working. **Must follow full marks in (a) apart from SC below.**		A1*
			(2)

Special case in (a) and (b) for those who do not have (a) in terms of *y* and *t* only:

$$t = x^2 \Rightarrow \frac{dt}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \ldots$$ **Scores M1.** $\ldots = 2x\frac{dy}{dt}$ **scores A0 in (a)(i)**

$$\frac{dy}{dx} = 2x\frac{dy}{dt} \Rightarrow \frac{d^2y}{dx^2} = 2\frac{dy}{dt} + 2x\frac{d^2y}{dt^2}\frac{dt}{dx} = 2\frac{dy}{dt} + 4x^2\frac{d^2y}{dt^2}$$ **Scores dM1A1A0 in (a)(ii)**

$$x\frac{d^2y}{dx^2} - \left(6x^2+1\right)\frac{dy}{dx} + 9x^3y = x^5 \Rightarrow t^{\frac{1}{2}}\left(2\frac{dy}{dt} + 4x^2\frac{d^2y}{dt^2}\right) - \left(6t+1\right)2x\frac{dy}{dt} + 9t^{\frac{3}{2}}y = t^{\frac{5}{2}}$$

$$\Rightarrow t^{\frac{1}{2}}\left(2\frac{dy}{dt} + 4t\frac{d^2y}{dt^2}\right) - \left(6t+1\right)2t^{\frac{1}{2}}\frac{dy}{dt} + 9t^{\frac{3}{2}}y = t^{\frac{5}{2}} \Rightarrow 4\frac{d^2y}{dt^2} - 12\frac{dy}{dt} + 9y = t^*$$ **Scores M1A1 in (b)**

Mark (c) and (d) together

(c)	$4m^2 - 12m + 9 = 0 \Rightarrow m = \frac{3}{2}$	Attempts to solve $4m^2 - 12m + 9 = 0$ Apply general guidance for solving a 3TQ if necessary.	M1
	$(y=)\mathrm{e}^{\frac{3}{2}t}\left(At+B\right)$	Correct CF. No need for "$y = \ldots$" Condone $(y=)\mathrm{e}^{\frac{3}{2}x}\left(Ax+B\right)$ here but must be in terms of t in the GS. Allow equivalents for the $\frac{3}{2}$.	A1
	$(y=)at+b \Rightarrow \frac{dy}{dt} = a \Rightarrow \frac{d^2y}{dt^2} = 0$ $\Rightarrow -12a + 9(at+b) = t$ Starts with the **correct** PI form and differentiates to obtain $\frac{dy}{dt} = a$ and $\frac{d^2y}{dt^2} = 0$ and substitutes. NB starting with a PI of $y = at$ is M0		M1
	$9a = 1 \Rightarrow a = \ldots$ $9b - 12a = 0 \Rightarrow b = \ldots$	Complete method to find a and b by comparing coefficients. **Depends on the previous method mark.**	dM1
	$y = \mathrm{e}^{\frac{3}{2}t}\left(At+B\right) + \frac{1}{9}t + \frac{4}{27}$	Correct GS including "$y = \ldots$" and must be in terms of t (no x's). Allow equivalent exact fractions for the constants.	A1
			(5)
(d)	$y = \mathrm{e}^{\frac{3}{2}x^2}\left(Ax^2+B\right) + \frac{1}{9}x^2 + \frac{4}{27}$ Correct equation including "$y = \ldots$" (follow through their answer to (c)). Allow equivalent exact fractions for the constants. For the ft, the answer to (c) must be in terms of t and the answer to (d) should be the same as (c) with t replaced with x^2. If there is no final answer to (c) you can award B1ft if the equation is correct in terms of x if it follows the previous work.		B1ft
			(1)
			Total 13